New mechanisms for reasoning and impacts accumulation for Rule Based Fuzzy Cognitive Maps

Zdanowicz, P & Petrovic, D

DOI 10.1109/TFUZZ.2017.2686363
ISSN 1063-6706
ESSN 1941-0034

Publisher: IEEE

New mechanisms for reasoning and impacts accumulation for Rule Based Fuzzy Cognitive Maps

Pawel Zdanowicz, Dobrila Petrovic

Coventry University, Faculty of Engineering, Environment and Computing, U.K.

Abstract – Rule Based Fuzzy Cognitive Maps (RBFCMs) have been developed for modelling non-monotonic, uncertain, cause-effect systems. However, the standard reasoning and impact accumulation mechanisms developed for RBFCMs assume that the level of variation that a fuzzy set represents is directly linked with the shape of the fuzzy set. It poses a big restriction on how the corresponding fuzzy sets have to be constructed. In this paper we propose a new reasoning and impact accumulation mechanisms which take into consideration standard semantics of fuzzy sets, where their uncertainty is measured by fuzziness. New type of complex fuzzy relationships and reasoning on them is introduced to model a joint impact of several causal nodes on one effect node. With these new mechanisms, RBFCMs become much more flexible, provide more means to capture complexity of real world systems and are less computational demanding than standard mechanisms. The advantages of the new RBFCMs are demonstrated using different examples and compared with standard mechanisms.

Index Terms – Fuzzy Cognitive Maps, Rule Based Cognitive Maps, Fuzzy Logic, Fuzzy Causal Relationship, Reasoning Mechanism

I. INTRODUCTION

Fuzzy Cognitive Maps (FCMs) were introduced in [1] to model complex relationships between concepts. A FCM is a graph where nodes represent concepts of a system, whereas the behaviour of the system is modelled by relationships among the nodes. There are two types of nodes; a node that has a causal influence on another node is the causal node, whereas a node that is impacted by this influence is an effect node. Relationships between the nodes in the FCM are represented using adjacency matrix $[a_{ij}]$, where a_{ij}=0 means that there is no causal relationship, $a_{ij} = (0,1]$ means that there is a positive relationship between nodes *i* and *j*, i.e., if node *i* is increased then node *j* will increase and if $a_{ij} = [-1,0)$, there is a negative relationship, i.e. if node *i* is increased then node *j* will decrease. Each node in the FCM has a value in the range [0, 1] or [-1, 1]. The value of the effect node is calculated by adding multiplied values of the corresponding causal nodes and degrees of causality - weights of the corresponding relationships.

The drawback of FCMs is linearity and monotony of a relationship between a causal node and an effect node. Regardless of how the causal node changes, its impact is multiplied by a single weight defining its relationship with the effect node. Therefore, the impact received by the effect node is linearly dependent on the value of the causal node and weight of the relationship. A relationship in an FCM, is monotonic and symmetrical about midpoint of the range of values that a causal node can take when the threshold function is used to calculate the value of the effect node, i.e. for range equal to [-1, 1], the relationship is symmetrical in point 0. Despite the limitations and simplicity of relationships FCMs could represent, they have become a widely used modelling tool.

An application specific modelling tool is Fuzzy Control System (FCS). In FCS, values of causal and effect nodes are expressed using fuzzy sets and relationships between them using IF-

THEN rules. This method is widely used in the engineering domain to model fuzzy controllers. In contrast to FCMs, fuzzy IF-THEN rules enable modelling of non-monotonic and non-symmetrical relationships among nodes. A crisp input into the system has to be fuzzified first and then appropriate rules, defining a relationship between two nodes, are fired. Impact received by the effect node is also fuzzy and has to be defuzzified. Most often applied method of inference in FCS has been Mamdani inference method [2]. As a result, the impact received by the effect node from two causal nodes is an average of two impacts, weighted with their firing strengths and the areas of the fuzzy sets.

Another step in evolution of fuzzy causal systems has been made by introduction of Rule Based Fuzzy Cognitive Maps (RBFCMs) by Tome and Carvalho [3]. As in FCS, values of nodes are represented using fuzzy variables and relationships between entities are expressed using fuzzy IF-THEN rules. The rules represent impact that variations or changes in causal nodes have on effect nodes. Variations and/or levels of nodes are modelled using linguistic terms, such as *Decreased, Increased, High, Low,* and so on. The main difference between FCS and RBFCMs is in their reasoning mechanisms and the way impacts received by the effect node are accumulated. For example, if the firing levels of two rules are the same and the impact is represented by two linguistic terms *Increased Little* and *Increased*, by applying Mamdani inference, the defuzzified output falls in between the two fuzzy sets. However, if these two rules are fired in an RBFCM, the result of accumulation of two impacts, *Increased Little* and *Increased*, is a fuzzy set that represents impact *More than Increased*. RBFCMs have been successfully used in a few applications, such as forest fire modelling [4], socio economical systems [5], fisherman behaviour [6] [7], defence [8], and student-centred education [9].

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

The main characteristic of the method proposed by Carvalho and Tome is that there is a link between a variation represented by the fuzzy set and the shape of the fuzzy set [3]. The greater the area and support of the fuzzy set is the greater the variation it represents. Carvalho and Tome proposed a set of predefined membership functions that can be used in an RBFCM and that ensures that the results obtained are correct. Consequently, experts are required to use the set of proposed membership functions and this reduces flexibility and applicability of the method. As one of the main aspects of modelling using fuzzy logic is possibility to represent experts' knowledge in a flexible way, in this paper we propose a new reasoning mechanism and a new mechanism for accumulation of impacts in RBFCMs. New approaches use standard semantics of fuzzy sets and improve RBFCM's flexibility by removing the constraint prevalent in the standard mechanisms that limit the range of membership functions that can be used by experts.

One of the main difficulties in constructing fuzzy cognitive models is elicitation of knowledge. It can be either built by experts, resulting in subjectivity of the model and facing difficulties in extracting knowledge for complex maps, or induced from historical data using learning algorithms. The latter became an important topic of research of FCMs in depth reviewed in [17].

Learning of FCMs can be grouped into two categories – Hebbian learning and evolutionary algorithms (EA). Hebbian learning based methods [18] require input, output and an initial set of weights to be defined prior to learning. The algorithm iteratively changes weights to minimize the difference between the output of the simulation and the desired values of nodes. An example of Hebbian learning based reasoning was proposed in [19], where a learning approach is used to change weights between nodes to improve stability of the system. The second category – EA, has received in the recent years much more attention than Hebbian learning. The EA encapsulate a broad range of population based, meta heuristic methods including memetic algorithms, which

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

combine multiple methods to improve local search capabilities of the EA [20]. The EA explore the search space and evaluate candidate solutions using a fitness function, which consist of desired set of output nodes' values and optionally other parameters. The advantage of the EA is that they do not require specifying the initial set of weights between concepts as those are generated randomly at the beginning of the learning process. An example of application of the EA is Dynamic Multiagent Genetic Algorithm, where elements of population are represented by agents that can interact with their neighbours to improve the population. The algorithm is used to reconstruct Gene Regulatory Network by building a FCM which represents relationships between 200 genes [21].

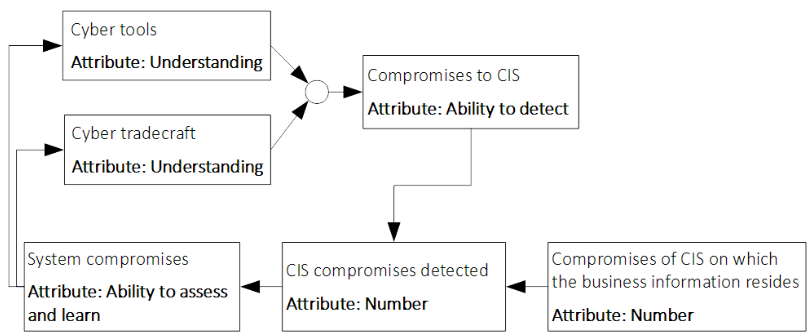
FCS learning methods can be divided into two categories: neural nets based and EAs. There are three aspects of the FCS that can be optimised: the shape of the membership functions, the rule base that defines a relationship between nodes and parameters of the inference used [22] [23].

The paper is organised as follows. In Section II, a case study is introduced and in the subsequent section semantics of fuzzy sets are discussed. In section IV, a standard RBFCM reasoning mechanism and its characteristics are presented followed by a description of a new reasoning mechanism proposed and benefits of employing it. Section V presents the standard RBFCM accumulation mechanism, its limitations and a solution to the problems by introducing a new method for accumulation of impacts. In Section VI, new types of relationships that can be defined and used in RBFCMs and the corresponding reasoning mechanism are explained. Conclusions are given in Section VII.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

II. CASE STUDY

One of the strengths of modelling using fuzzy logic is the ability to model systems on the higher level of abstraction and describing concepts that are difficult to represent using precise mathematical relationships. An example of such system is a cyber defence system of an enterprise. One of its subsystems is detection of compromises to Communication and Information Systems within organisation. Detection of compromises is one of the key aspects of providing security of information that is crucial for most organisations. This subsystem can be modelled using RBFCM as presented in Figure 1.



CIS – Communication and Information Systems

Fig. 1 An RBFCM for sybsystem of cyber defence - compromises detection

Some of the elements of the detection system are easily measurable, i.e. Number of CIS compromises detected. On the other hand, some elements, such as: Understanding Cyber tools and tradecrafts are concepts representing human capabilities that are difficult to quantify, and, therefore, relationships between them are difficult to define. In the presented model relationships between nodes are defined using fuzzy IF-THEN rules, i.e. IF Ability to detect compromises to CIS is *Increased Much* THEN Number of CIS compromises detected is *Increased*. Fuzzy terms used to define these relationships are modelled using membership functions. As in [3], we use seven values of variations in causal nodes and consequent variations in effect nodes, *Decreased Much* (DM), *Decreased* (D), *Decreased Little* (DL), *Maintained* (M), *Increased Little* (IL),

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

Increased (I), *Increased Much* (IM). However, the corresponding fuzzy sets are defined by cyber defence experts, in a different way compared to [3]. They are given in Figure 2, where x axis represents percentage of variations. Examples from this case study are used to explain improvements to the RBFCM mechanisms proposed in this paper.

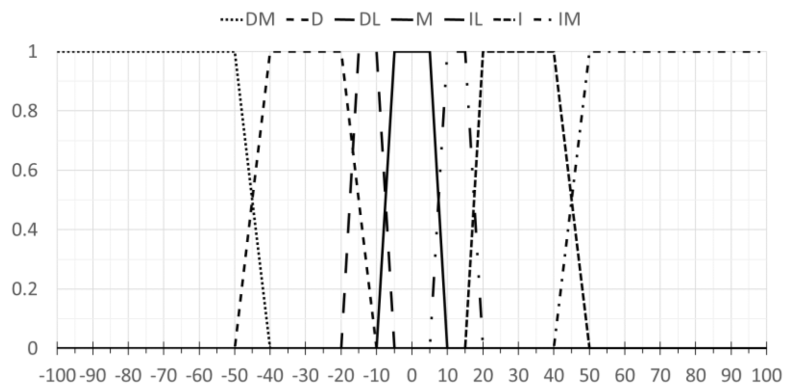


Fig. 2 Linguistic terms that represent variations in causal and effect nodes

III. SEMANTICS OF FUZZY SETS

Two main types of fuzzy sets used in RBFCMs represent variation of an attribute which describes a node (*small increase, big increase,* etc.) and level of attribute (*low level, high level,* etc.). The standard interpretation of these fuzzy sets is one proposed by Bellman and Zadeh [10], where membership function is defining a degree of preference or belief, $\mu(x)$, of object x belonging to fuzzy set A over other objects, where $\mu(x)=0$ means that the object does not belong to fuzzy set A, whereas $\mu(x)=1$ defines the full membership. In this paper, trapezoidal membership functions are considered but any type of membership functions can be used provided they fulfil the following two requirements [11]:

1) Two consecutive membership functions defining variations overlap and cross in the point x, where $\mu(x)=0.5$ and
2) The sum of all membership degrees for a given point x is equal to 1. (1)

A reasoning mechanism and accumulation of impacts in RBFCMs proposed in [3], have been built assuming specific relationships between degrees of variations of fuzzy sets and their areas and positions on the universe of discourse [11]. This condition greatly affects the way fuzzy sets of degrees of variation (or levels) have to be constructed and limits expert's flexibility when defining these fuzzy sets. The fuzzy set A represents a greater variation than fuzzy set B when following condition is fulfilled:

$$\forall A, B \in \mathcal{F}(X),\ A > B \Leftrightarrow support_A > support_B \wedge Area_A > Area_B \qquad (2)$$

where $\mathcal{F}(X)$ represents fuzzy sets defined in the universe of discourse.

In Figure 3 (a) and (b), fuzzy set A, *Increased*, represents a greater variation than fuzzy set B, *Increased Little*, as it lays further from the beginning of the scale (point 0). In the case presented in Figure 3 (b), fuzzy set B has a bigger area than fuzzy set A, and, therefore, these fuzzy sets cannot be used to model knowledge on degrees of variations in an RBFCM.

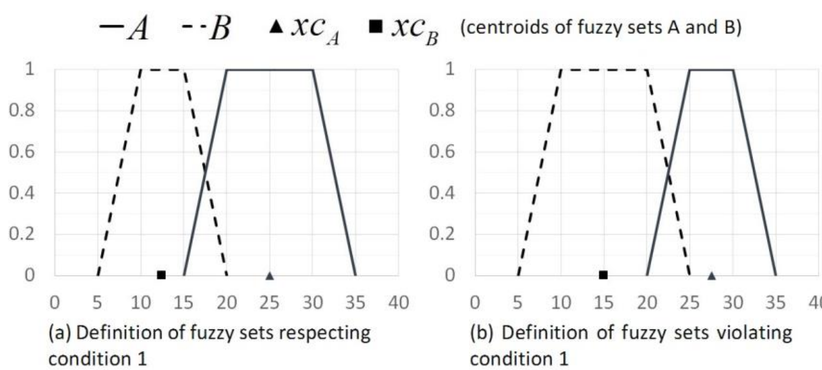


Fig. 3 An example of fuzzy sets which represent variations (or levels)

Fuzzy sets presented in Figure 2, defined by the cyber defence experts, do not fulfil the condition (2) as fuzzy set *Maintained* has greater area than fuzzy set representing greater variation *Increased Little*, therefore cannot be used in a standard RBFCM.

In the new RBFCM reasoning mechanism, this condition has been removed to allow using standard understanding of the fuzzy sets' semantics, where the area, support, core and inner and outer bases are used to represent uncertainty modelled by the fuzzy set, presented in Figure 4.

Metric that we use to measure uncertainty in RBFCMs is degree of fuzziness. Degree of fuzziness defines how different from its complement the fuzzy set is. The more different from its complement the fuzzy set is, the less fuzzy it is; the fuzziness of a crisp fuzzy set is equal to 0. Degree of fuzziness can be used to assess uncertainty of experts. Different functions can be used to measure fuzziness of a fuzzy set. In this paper, we use Index of fuzziness [12]. Index of fuzziness of fuzzy sets A for Minkowski class of distances [13] is defined as:

$$f_{c,\mathrm{w}}(A) = (d-a)^{1/w} - \left(\int_a^d \delta_{c,A}^{w}(x)\,dx\right)^{1/w}, w \in [1,\infty) \tag{3}$$

where, $\delta_{c,A}^{w}$ is the distance between fuzzy set A and its complement $c(A)$ raised to the power of w,

a and d are the minimum and maximum of support of fuzzy set A, respectively.

In this paper, Hamming distance is used, where $w=1$:

$$f_{c,1}(A) = (d-a) - \int_a^d \delta_{c,A}^{1}(x)\,dx \tag{4}$$

where, $\delta_{c,A}^{1}(x) = \left|\mu_A(x) - \mu_{c(A)}(x)\right|$ is the Hamming distance between degree of belief of fuzzy set A and its complement $c(A)$, for a given point x, and

$\mu_{c(A)}(x) = 1 - \mu_A(x)$.

It can be observed that for any trapezoidal membership function:

$$\int_a^d \delta_{c,A}^{1}(x)\,dx = (d-a) - \left(\frac{bi_A}{2} + \frac{bo_A}{2}\right) \tag{5}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

represented as the shaded area in Figure 4.

Therefore, Index of fuzziness is

$$f_{c,1}(A)=(d-a)-\int_a^d \delta_{c,A}^1(x)\,dx=(d-a)-(d-a)+\left(\frac{bi_A}{2}+\frac{bo_A}{2}\right)=\frac{bi_A+bo_A}{2} \tag{6}$$

and normalized fuzziness is:

$$\hat{f}_{c,1}(A)=\frac{\frac{bi_A+bo_A}{2}}{(d-a)}=\frac{bi_A+bo_A}{2\ \mathrm{support}_A}=\frac{bi_A+bo_A}{2\ (bi_A+bo_A+\mathrm{core}_A)} \tag{7}$$

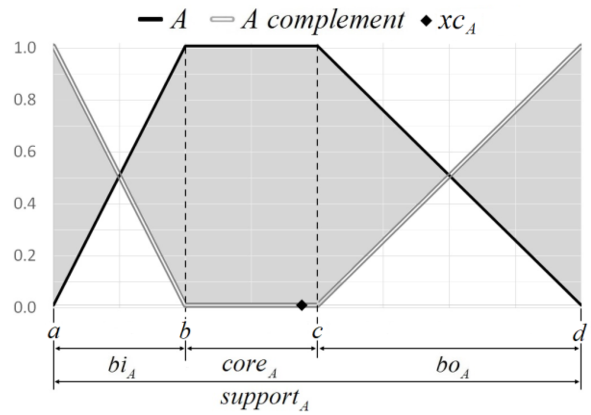


Fig. 4 Hamming distance between fuzzy set A and its complement

Factors that impact the normalised fuzziness are the length of the core, inner and outer bases of the fuzzy set. The greater the inner and outer bases are, for a given support, the fuzzier the fuzzy set is. When inner and outer bases are equal to 0, i.e. the fuzzy set is crisp, there is no fuzziness. The same relation, an increase of fuzziness, can be observed when the size of the inner and outer bases remains unchanged, but the size of core and support become smaller. If the length of core is equal to 0, i.e. it is 1 point, the fuzziness is the highest, equal to 0.5. The exception is a singleton fuzzy set, where fuzziness is the lowest.

The reasoning mechanism proposed in this paper is defined in such a way as to preserve this metric and use it in reasoning instead of the area of the fuzzy set.

IV. RBFCM REASONING MECHANISMS

A. *Standard RBFCM reasoning mechanism*

A simple fuzzy causal relationship, FCR, between two nodes, causal Node X and effect Node Z is defined using IF-THEN rules where the antecedent of a rule defines the value of Node X and the consequent of the rule specifies the value of Node Z. The FCR can be modelled by up to *N* rules, where *N* is the number of fuzzy terms that the causal node can have; in this case, seven fuzzy sets are used, *N* = 7. The FCR is defined as follows:

R_i: IF Node X is A_i THEN Node Z is C_i, i=1…N

The reasoning algorithm developed in [14] assumes that the conditions (1) are satisfied. Consequently, either one or two rules R_i, $i = 1,\ldots,N$ can be fired.

To explain the standard reasoning mechanism, let us consider a relationship between two nodes Number of CIS compromises detected and Ability to assess and learn from system compromises represented in Figure 1. Fuzzy sets modelling variations are defined in Figure 5, where membership functions are defined respecting condition (2).

Let us assume that the input into the node Number of CIS compromises detected is decreased and falls in between supports of two fuzzy sets *Decreased Little and Maintained*. This triggers two rules *i* and *j*, with degrees of belief m_{A_i} and m_{A_j}, respectively, where $m_{A_i} + m_{A_j} = 1$.

R_i: IF Number of CIS compromises detected is *Decreased Little* (m_{A_i})
THEN Ability to assess and learn from system compromises is *Maintained* (m_{C_i})
R_j: IF Number of CIS compromises detected is *Maintained* (m_{A_j})
THEN Ability to assess and learn from system compromises is *Increased Little* (m_{C_j})

where degrees of belief of the antecedent and the consequent are given in the brackets.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

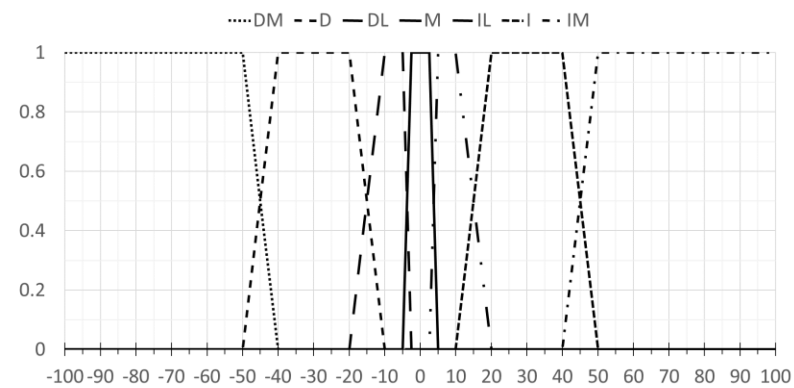


Fig. 5 Definition of variations' fuzzy sets respecting condition (2)

To handle the case when two rules are fired, an algorithm for combining the impact of two fired rules was proposed [14]. The process of reasoning on these two rules is presented in Figure 6, where nodes X and Y represent nodes Number of CIS compromises detected and Ability to assess and learn from system compromises, fuzzy set A_i and A_j are fuzzy set *Decreased Little* and *Maintained*, C_i and C_j are fuzzy set *Maintained* and *Increased Little*, respectively. First, fuzzy sets of the consequent of fired rules, in this example, *Maintained*, C_i, and *Increased Little*, C_j, are "cut" into C'_i and C'_j, respectively, corresponding to the firing levels m_i and m_j. Union U of these two "cut" fuzzy sets is determined using Max-product method as follows:

$$m_i = m_{A_i}$$
$$C'_i = C_i(z) \cdot m_i$$
$$m_j = m_{A_j}$$
$$C'_j = C_j(z) \cdot m_j$$

$$U(z) = \max[C'_i(z), C'_j(z)]$$

where m_{A_i} is firing level of fuzzy set A_i, m_{A_j} is firing level of fuzzy set A_j.

Centroid xc_U of union U represents the crisp impact received by Node Z - Ability to assess and learn from system compromises. The next step of the process is to determine a new fuzzy set

Causal Output Set (COS_U), which represents combined variations and preserves the trapezoidal shape of the two consequent fuzzy sets. COS_U is determined in such a way as to satisfy the following condition:

$$Area_{COS_U} = Area_U \tag{8}$$

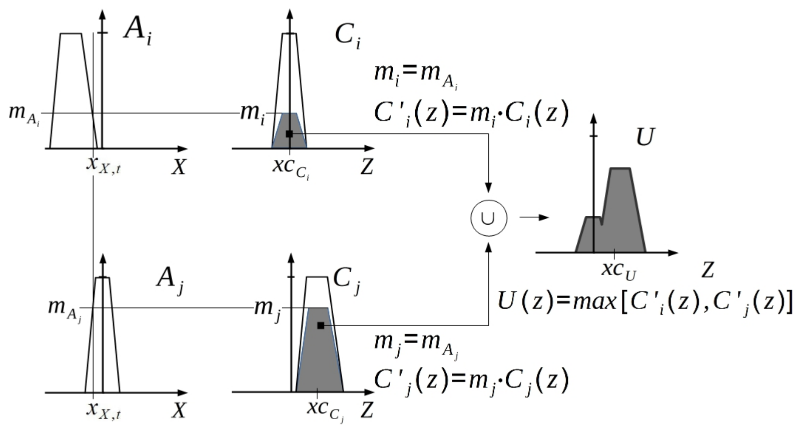


Fig. 6 Reasoning mechanism when two rules are fired

The values of $core_{COS_U}$ and bi_{COS_U} are calculated based on the distance between defuzzified consequent fuzzy sets C_i and C_j, xc_{C_i} and xc_{C_j}, respectively, and defuzzified union of the two “cut” fuzzy sets, xc_U, as follows:

$$core_{COS_U} = min\left\{core_{C_i}, core_{C_j}\right\} + \left|\frac{xc_U - xc_{C_i}}{xc_{C_j} - xc_{C_i}} \times \left(core_{C_j} - core_{C_i}\right)\right| \tag{9}$$

$$bi_{COS_U} = min\left\{bi_{C_i}, bi_{C_j}\right\} + \left|\frac{xc_U - xc_{C_i}}{xc_{C_j} - xc_{C_i}} \times \left(bi_{C_j} - bi_{C_i}\right)\right| \tag{10}$$

where defuzzification of fuzzy set is carried out using the Centroid method [16].

bi_{COS_U} and $core_{COS_U}$ are used to calculate the remaining characteristics of COS_U : $support_{COS_U}$ and bo_{COS_U} , as follows:

$$bo_{COS_U} = 2\,Area_{COS_U} - (2\,core_{COS_U} + bi_{COS_U}) \tag{11}$$

$$support_{COS_U} = bi_{COS_U} + core_{COS_U} + bo_{COS_U} \tag{12}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

The centroid of new fuzzy set COS_U is calculated using the Centroid defuzzification method and, then, the fuzzy set COS_U is shifted so that $xc_{COS_U} = xc_U$ as in Figure 7. Fuzzy set COS_U represents the impact of the causal node Number of CIS compromises detected on the effect node Ability to assess and learn from system compromises.

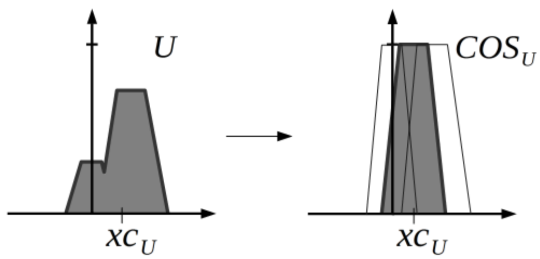


Fig. 7 Transformation of union U into the fuzzy set COS_U

If the input from the user falls within a core of one of the fuzzy sets, then only one corresponding rule is fired with firing level equal to 1. As a result, fuzzy set COS_U becomes the consequent fuzzy set of the fired rule.

B. A new reasoning mechanism

In this paper, a new reasoning mechanism for COS_U calculation, when more than one rule is fired, is proposed. The algorithm is focused on maintaining uncertainty of the fired fuzzy sets rather than the relationship between $Area_U$ and $Area_{COS_U}$. Therefore, condition (2) does not have to be respected when membership functions of fuzzy variations are defined. The new method is based on the idea that COS_U should have a shape similar to the consequent fuzzy set to which centroid the centroid of COS_U is closer to. In this way, the fuzziness of the COS_U will be similar to the fuzziness of the nearer consequent fuzzy set.

Let us consider the example of relationship between two nodes Number of CIS compromises detected and Ability to assess and learn from system compromises, as in the previous section. The two rules are fired as follows:

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

R_i : IF Number of CIS compromises detected is *Decreased Little* (m_{A_i})
THEN Ability to assess and learn from system compromises is *Maintained* (m_{C_i})

R_j : IF Number of CIS compromises detected is *Maintained* (m_{A_j})
THEN Ability to assess and learn from system compromises is *Increased Little* (m_{C_j})

where m_{A_i} is firing level of rule *i* and $m_{C_i} = m_{A_i}$,

m_{A_j} is firing level of rule *j* and $m_{C_j} = m_{A_j}$, and $m_{A_i} + m_{A_j} = 1$.

It can be observed that the centroid of union U of two "cut" fuzzy sets C'_i and C'_j, takes the following value (Figure 8 (a) and (b)):

$xc_U = xc_{C_i}$, when $m_{C_i} = 1$ and $m_{C_j} = 0$,

$xc_U = xc_{C_j}$, when $m_{C_i} = 0$ and $m_{C_j} = 1$,

$xc_{C_i} < xc_U < xc_{C_j}$ when $m_{C_i} \in (0,1)$ and $m_{C_j} \in (0,1)$,

where $m_{C_i} + m_{C_j} = 1$

For example, assume that $m_{C_i} > m_{C_j}$ (Figure 8 (a)). Then:

$dist_{C_i} < dist_{C_j}$

where $dist_{C_i} = \left| xc_U - xc_{C_i} \right|$ is the distance between centroid of C_i, xc_{C_i}, and centroid of U, xc_U,

$dist_{C_j} = \left| xc_U - xc_{C_j} \right|$ is the distance between centroid of C_j, xc_{C_j}, and centroid of U, xc_U.

The importance of impact of a "cut" consequent fuzzy set is represented as its inverted distance from union U of the "cut" consequent fuzzy sets; the nearer the centroid of the "cut" consequent fuzzy set to centroid of U is, the higher its impact is. In Figure 8 (a), centroid xc_{C_i} of C_i is nearer to centroid xc_U of union U and, therefore, its impact on COS_U is higher, while in Figure 8 (b), centroid xc_{C_j} is nearer to centroid xc_U and, therefore, impact of C_j is higher.

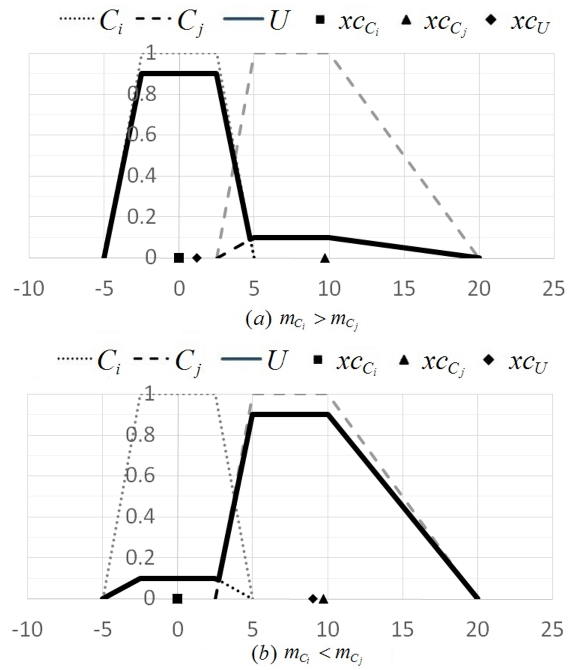


Fig. 8 Relation between firing levels m_{C_i}, m_{C_j} and centroid of union U of the cut fuzzy sets C_i' and C_j'

The impacts are normalised using normalising factor c as follows:

$$c = \frac{1}{\frac{1}{dist_{C_i}} + \frac{1}{dist_{C_j}}} \tag{13}$$

COS_U is determined using the following formulas:

$$core_{COS_U} = c\left(\frac{core_{C_i}}{dist_{C_i}} + \frac{core_{C_j}}{dist_{C_j}}\right) \tag{14}$$

$$bi_{COS_U} = c\left(\frac{bi_{C_i}}{dist_{C_i}} + \frac{bi_{C_j}}{dist_{C_j}}\right) \tag{15}$$

$$bo_{COS_U} = c\left(\frac{bo_{C_i}}{dist_{C_i}} + \frac{bo_{C_j}}{dist_{C_j}}\right) \tag{16}$$

$$support_{COS_U} = bi_{COS_U} + core_{COS_U} + bo_{COS_U} \tag{17}$$

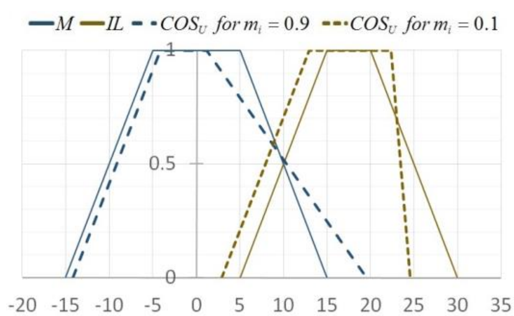
Once the characteristics of COS_U are obtained, it is shifted so that $xc_{COS_U} = xc_U$.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

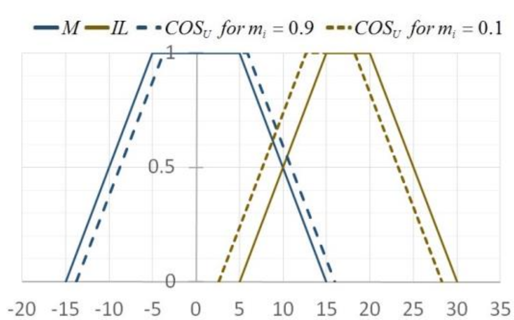
C. Characteristics of the reasoning mechanisms

(1) Preservation of fuzziness

In some applications, the expert can define fuzzy sets violating condition (2). An example of such two fuzzy sets, *Maintained* and *Increased Little*, is given in Figure 9 (a) and (b), where the support and area of fuzzy set *Maintained* are greater than the respective values of fuzzy set *Increased Little*. Let us assume that both fuzzy sets *Maintained* and *Increased Little* are fired with degrees m_{C_i} and $m_{C_j} = 1 - m_{C_i}$, respectively. For firing levels m_{C_i} approaching 1, the centroid of the union U of "cut" fuzzy sets *Maintained* and *Increased Little* is approaching the centroid of fuzzy set *Maintained*; therefore, the size of $core_{COS_U}$ is also approaching the size of the core of fuzzy set *Maintained*. On the other hand, when firing level m_{C_i} is approaching 0, the centroid of union U is approaching the centroid of fuzzy set *Increased Little* and, in consequence, the size of $core_{COS_U}$ is approaching the size of the core of fuzzy set *Increased Little* (Figure 9 (b)).



(a) Fuzzy set COS_U, for firing levels m_i=0.9 and m_i=0.1 when the standard reasoning mechanism is used



(b) Fuzzy set COS_U, for firing levels m_i=0.9 and m_i=0.1 when the new reasoning mechanism is used

Fig. 9 Comparison of the new and standard reasoning mechanisms

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

In each Figure 9 (a) and (b), two fuzzy sets COS_U are shown; one, resulting from firing fuzzy set *Maintained* with degree of belief $m_{C_i} = 0.9$ and fuzzy set *Increased Little* with degree of belief $m_{C_j} = 0.1$, the second, resulting from firing fuzzy set *Maintained* with degree of belief $m_{C_i} = 0.1$ and fuzzy set *Increased Little* with degree of belief $m_{C_j} = 0.9$. It can be observed that when the new reasoning mechanism is used, the shape of fuzzy set COS_U resembles the shape of the consequent fuzzy set it is closer to, *Maintained* or *Increased Little*, and maintains the fuzziness of the consequent fuzzy set it is closer to. When the standard reasoning mechanism is used, the shape and fuzziness of COS_U are different from fuzzy sets *Maintained* and *Increased Little*.

(2) Normality and convexity of the COS_U

The standard reasoning mechanism is based on the idea that the area and variation represented by the fuzzy set are related, i.e. larger variations are represented by fuzzy sets with larger supports and areas. However, in some cases, this approach can result in COS_U being a trapezoid with a core longer than its support. Therefore, the obtained result is no longer a fuzzy number.

To illustrate this problem, let us consider a relationship between Node X and Node Z with membership functions defined as in Figure 5. Let us assume that for any input between 10 and 20, the following two rules will be fired:

R_i: IF Node X is *Increased Little* (m_{A_i}) THEN Node Z is *Increased Little* (m_{C_i})
R_j: IF Node X is *Increased* $(1 - m_{A_i})$ THEN Node Z is *Increased Much* $(1 - m_{C_i})$
where $m_{C_i} = m_{A_i}$.

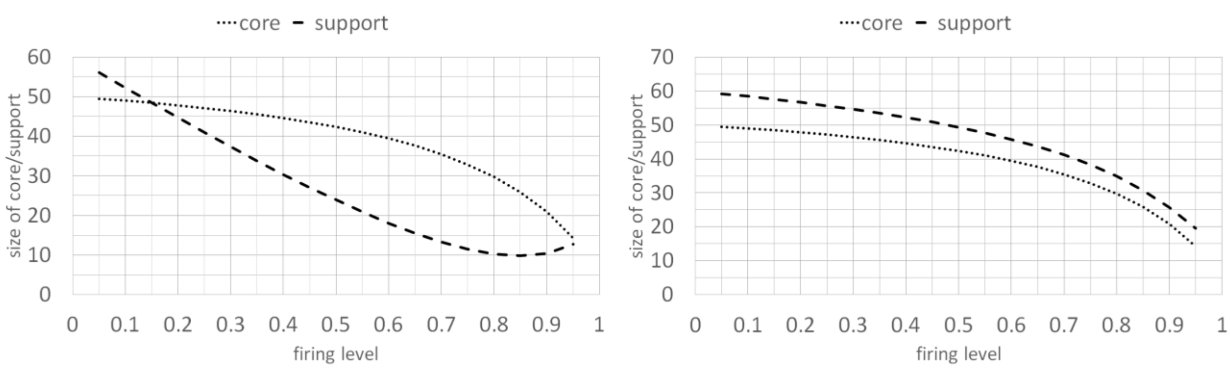
The size of the core of the fuzzy set *Increased Little* is $core_{IL} = 5$ and its area is $Area_{IL} = 11.25$, whereas the core of fuzzy set *Increased Much* is $core_{IM} = 50$ and its area is $Area_{IM} = 55$. For example, for the input 13 the firing level m_{A_i} of the first rule is equal to

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

$m_{A_i} = 0.3$ and, therefore, of the second rule is $1 - m_{A_i} = 0.7$. Using formulas (8) to (12), COS_U is calculated as $Area_U = 41.8$, $core_{COS_U} = 46.3$, $bi_{COS_U} = 2.6$, $bo_{COS_U} = -11.6$ and $support_{COS_U} =$ 37.3. As it can be seen, the support is shorter than the core of COS_U .

In Figure 10 (a) the relationship between the area, core and support of COS_U with respect to different firing levels m_{A_i} of the two rules given in the Example is presented. For firing levels m_{A_i} in the interval [0.15, 0.95], the size of the $core_{COS_U}$ is greater than the size of the $support_{COS_U}$, and, therefore, these trapezoidal functions cannot be used as membership functions for COS_U .

To avoid this problem, in the reasoning mechanism proposed, the relation between the area of the union U of the "cut" consequent fuzzy sets and the area of COS_U is removed.



(a) Standard reasoning mechanism (b) New reasoning mechanism

Fig. 10 Relationship between area, core and support of COS_U , for different firing levels of the two rules

The results of using the new mechanism, for the discussed example are presented in Figure 10 (b). When compared with the results of the standard RBFCM reasoning mechanism, it can be observed that the size of $core_{COS_U}$ is smaller than the size of $support_{COS_U}$ for all firing levels. The new reasoning mechanism calculates the same size of the core, as the standard reasoning, but as it does not preserve the areas of U and COS_U , it determines different area and support of COS_U than the standard reasoning mechanism. It calculates $core_{COS_U}$ and $support_{COS_U}$ in such a way that

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

$core_{COS_U} < support_{COS_U}$. Therefore, the resulting COS_U can be considered as a convex and normal trapezoidal fuzzy set.

V. ACCUMULATION OF IMPACTS

A. *Standard RBFCM accumulation mechanism*

Accumulation process is carried out when more than one causal nodes are affecting an effect node. Let us consider a case where two causal nodes: Ability to detect compromises to CIS and Number of compromises of CIS on which information resides are in a causal relationship with one effect node, Number of CIS compromises detected, as shown in Figure 1. If, for example, Ability to detect compromises to CIS is *Increased* and causes Number of CIS compromises detected to *Increase* and Number of compromises of CIS on which information resides is also *Increased* and causes Number of CIS compromises detected to *Increase Little*, it seems appropriate that Number of CIS compromises detected should change to *More than Increase*. However, the standard inference mechanisms, such as Mamdani [2] or Sugeno [15], will result in Number of CIS compromises detected changing somewhere between value *Increased Little* and *Increased* depending on the strength of each impact. Therefore, a different mechanism for accumulation of impacts in RBFCMs is required, as proposed in [14].

The accumulation of impacts mechanism requires shifting the fuzzy set representing a lower variation towards the fuzzy set representing a greater variation. Before shifting and accumulation are conducted, it is necessary to determine which fuzzy set represents lower variation. Fuzzy set A represents lower variation then fuzzy set B when:

$$min(core_A) < min(core_B)$$

Fuzzy set A is shifted towards fuzzy set B until the following condition is met (Figure 11 (a)):

$$min\left(support_{A_{shifted}}\right)=min\left(core_B\right)$$

Therefore, fuzzy set A is shifted towards fuzzy set B by distance:

$$shift_A = min\left(core_B\right) - min\left(support_A\right)$$

The standard RBFCM accumulation process is a discrete, recursive algorithm. It accumulates degrees of belief of every point x as the sum of the corresponding degrees of belief of two fuzzy sets, shifted A and B; if it is greater than 1, then the surplus in degrees is carried forward towards points where the sum of degrees of belief is lower than 1. The resulting Variation Output Set, VOS_+, representing an accumulated value of positive impacts, is calculated as follows:

$$\mu_{VOS_+}(x) = min\left\{1, \mu_B(x_i) + \mu_A(x_i - shift_A) + carry(x_{i-1})\right\} \tag{18}$$

$$carry(x_i) = max\left\{0, \mu_B(x_i) + \mu_A(x_i - shift_A) + carry(x_{i-1}) - 1\right\}$$
$$carry(x_{-1}) = 0 \tag{19}$$

The result of accumulating two impacts A and B, presented in Figure 11 (a), is shown in Figure 11 (b). The centroid of xc_{VOS_+} is calculated using the Centroid defuzzification method [16].

Described algorithm performs accumulation of positive impacts. All accumulated negative impacts form negative Variation Output Set, VOS_- In order to accumulate negative impacts, when fuzzy sets A and B represent negative variations, described algorithm needs to be altered accordingly, i.e. fuzzy set A is shifted to fuzzy set B until the following condition is fullfiled:

$$\max\left(support_{A_{shifted}}\right)=\max\left(core_B\right).$$

After accumulation of both positive and negative impacts, the total variation received by the effect node is determined as the sum of the centroids xc_{VOS_+} and xc_{VOS_-} of the two variation output sets, positive VOS_+ and negative VOS_- respectively, weighted by their respective areas:

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

$$\text{variation} = \frac{xc_{VOS_+} Area_{VOS_+} + xc_{VOS_-} Area_{VOS_-}}{Area_{VOS_+} + Area_{VOS_-}} \tag{20}$$

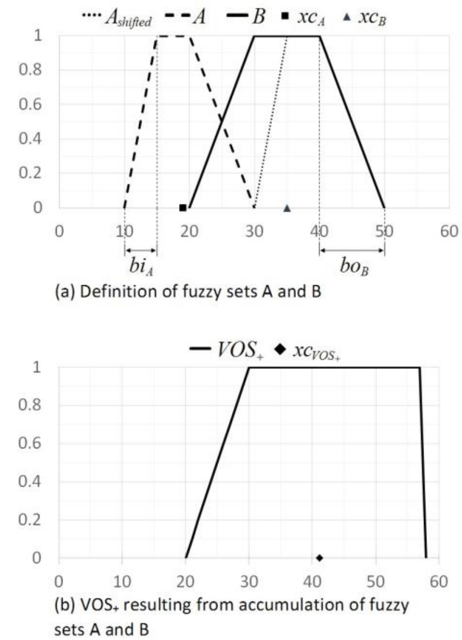


Fig. 11 Accumulation of impacts using the standard accumulation mechanism

The only characteristic of the involved fuzzy sets that is preserved during this shifting approach is the inner base of the fuzzy set representing the greater variation. In the given example, the inner base of VOS_+ is the same as the inner base of fuzzy set B. The information about position of the shifted fuzzy set is lost.

B. A new accumulation mechanism

The standard accumulation mechanism relies on the relationship between linguistic variation and the shape of the fuzzy set expressed in condition (2). We are proposing a new method of accumulating impacts when variations of a causal and effect nodes are defined in a flexible way, i.e. the linguistic variation values are not linked with the shapes and areas of the corresponding fuzzy sets. The new method accumulates impacts considering fuzziness of fuzzy sets.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

Centroid xc_{VOS_+} of fuzzy set VOS_+, is calculated considering centroids of fuzzy sets A and B, xc_A and xc_B, as follows:

$$xc_{vos_+} = (xc_B + xc_A)p \tag{21}$$

$$p = 1 - \frac{\hat{f}(A) + \hat{f}(B)}{2s} \tag{22}$$

Parameter p depends on the fuzziness of fuzzy sets involved in the accumulation. The higher the fuzziness $\hat{f}(A)$ and/or $\hat{f}(B)$ is, the smaller the parameter p and xc_{vos_+} are. Parameter p takes the maximum value 1 when sets A and B are crisp sets or singletons i.e., when $\hat{f}(A) = \hat{f}(B) = 0$. As a result, xc_{vos_+} is equal to the arithmetic sum of centroids of the respective sets and impacts are fully accumulated. Parameter p takes the minimum value 0.5 when both fuzzy sets A and B are triangular fuzzy sets, $\hat{f}(A) = \hat{f}(B) = 0.5$ and $s = 1$.

Parameter s is introduced to increase the impact of accumulation. The higher the value of parameter s is, the stronger the accumulation of impacts. Table I summarizes the impact of the parameter s on the parameter p.

Fuzzy set VOS_+ is calculated using the standard summation of fuzzy sets A and B, $VOS_+ = U = A + B$ (Figure 12). Characteristics of VOS_+ are calculated as follows:

$$core_{VOS_+} = core_B + core_A \tag{23}$$

$$bi_{VOS_+} = bi_A + bi_B \tag{24}$$

$$bo_{VOS_+} = bo_A + bo_B \tag{25}$$

$$support_{VOS_+} = bi_{VOS_+} + core_{VOS_+} + bo_{VOS_+} \tag{26}$$

After they are calculated the fuzzy set is shifted so that the centroid of VOS_+ is equal to centroid calculated using (21).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

TABLE I
IMPACT OF PARAMETER S ON THE STRENGTH OF ACCUMULATION

s	$p_{\min}$	$p_{\max}$
1	0.5	1
2	0.75	1
4	0.875	1
10	0.95	1

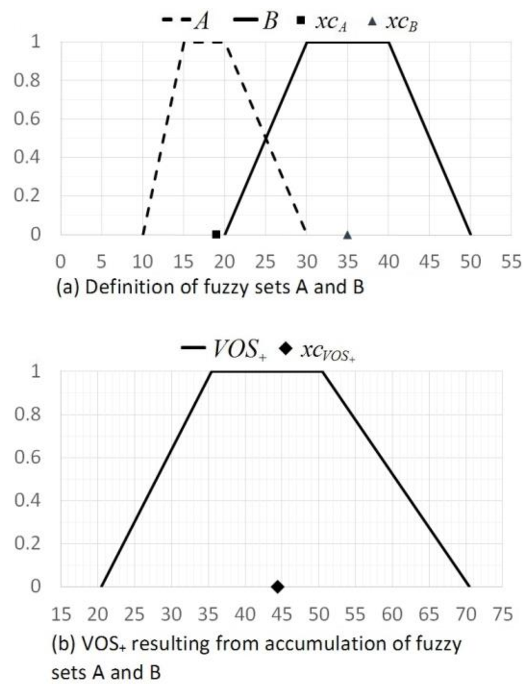


Fig. 12 Accumulation of impacts using the new accumulation method

Negative impacts are accumulated in the similar way to the accumulation of positive impacts.

We are proposing a new approach to variation calculations which considers fuzziness of positive VOS_+ and negative VOS_- impacts received:

$$\text{variation} = \begin{cases} xc_{VOS_-} & \text{if } VOS_+ = \varnothing \\ xc_{VOS_+} & \text{if } VOS_- = \varnothing \\ \dfrac{xc_{VOS_+}\left(1-\hat{f}\left(VOS_+\right)\right)+xc_{VOS_-}\left(1-\hat{f}(VOS_-)\right)}{2-\left(\hat{f}\left(VOS_+\right)+\hat{f}(VOS_-)\right)} & \text{otherwise} \end{cases} \tag{27}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

Impacts VOS_+ and VOS_- are weighted with their levels of fuzziness. The less fuzziness in fuzzy set VOS_+ or VOS_- is, the more important in accumulation of impacts it is.

C. Characteristics of the accumulation mechanisms

(1) Commutativity and associativity

In an RBFCM, an effect node can be impacted by more than two causal nodes, therefore the possibility of receiving, simultaneously, more than two fuzzy impacts exist. To assure that the order of receiving impacts is not predominant, accumulation of impacts mechanism should be commutative and associative.

The standard accumulation of impacts mechanism is commutative; regardless whether impact A or B is received first, the result of accumulation of impacts A and B is always the same as the fuzzy set representing the smaller variation is shifted towards the other one representing the greater variation. The fact that the inner slope of the fuzzy set representing the greatest variation becomes the inner slope of the VOS_+ causes the standard accumulation mechanism to be non-associative; the order of the impacts received does matter when there are three or more impacts received simultaneously.

On the other hand, the accumulation of impacts mechanism proposed, is both commutative and associative, as it considers all the impacts received by an effect node in a single operation that is both commutative and associative (see formulas (21) – (26)).

(2) Accumulation sensitivity

Let us consider three examples to demonstrate differences between the standard and the new mechanisms for accumulation of impacts.

In Example 1, definitions of fuzzy sets A – *Increased* and B – *Increased Little* violates condition (2), as presented in Figure 13 (a). Let us assume that an effect node receives both

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

impacts A and B, and $xc_A = 15$, $Area_A = 15$, $xc_B = 27.5$ and $Area_B = 10$, respectively. In Example 2, presented in Figure 14 (a), the effect node receives two identical impacts *Increased* and $xc_A = xc_B = 27.5$ and $Area_A = Area_B = 10$.

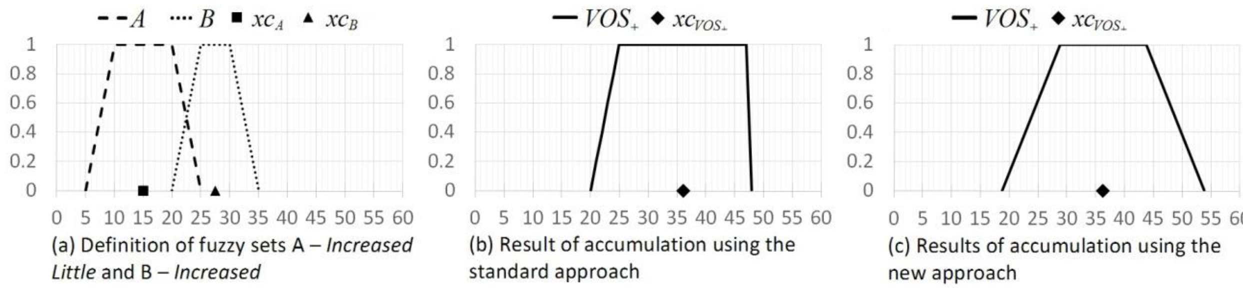


Fig. 13 Comparison of results of the two mechanisms for accumulation of impacts in Example 1

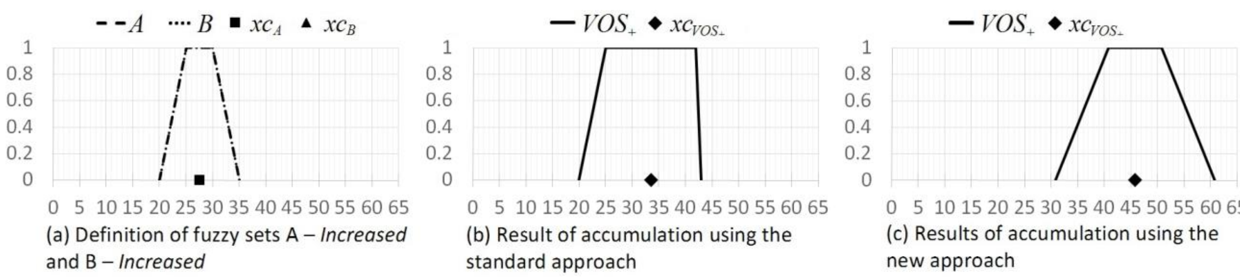


Fig. 14 Comparison of results of the two mechanisms for accumulation of impacts in Example 2

Table II shows the results of the two accumulation mechanisms. Parameter s in the new mechanism is arbitrarily set to $s = 4$ to increase the impact of accumulation. As it can be seen, if the standard RBFCM accumulation is used, the combined effect of two impacts *Increased* in Example 2 is equal to 32.45. However, it is smaller than the combined effect of two impacts *Increased* and *Increased Little*, in Example 1, which is equal to 36.07. This means that receiving two impacts representing a larger variation, such as *Increased*, results in a smaller accumulated value than if one smaller, *Increased Little*, and one larger impact, *Increased*, are received and accumulated using the standard accumulation algorithm. It is due to the larger area of the fuzzy set *Increased Little* than the area of the fuzzy set *Increased.* On the other hand, the new mechanism accumulates impacts based on their fuzziness levels, rather than areas. Therefore, in

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

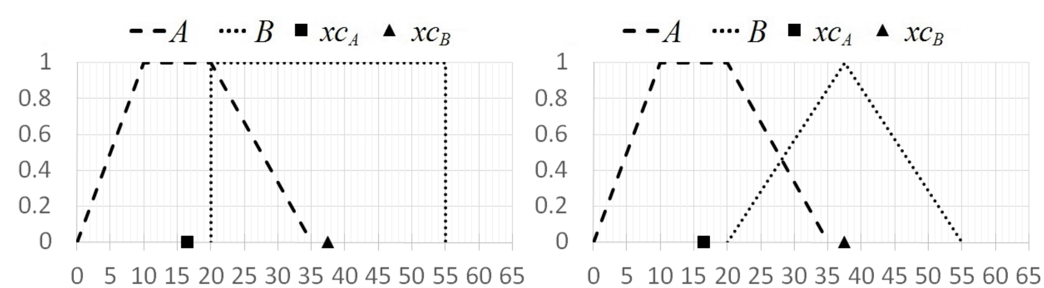
Example 1, the combined effect of *Increased* and *Increased Little* is equal to 36.3, and the combined effect of two impacts *Increased* is higher, equal to 45.83, when $s=4$.

TABLE II

COMPARISON OF RESULTS OBTAINED USING THE TWO ACCUMULATION MECHANISMS

	Example 1	Example 2	Example 3	Example 4
xc_A	15	27.5	16.5	16.5
xc_B	27.5	27.5	37.5	37.5
xc_{VOS_+} standard mechanism	36.1	32.5	48.7	49.1
xc_{VOS_+} new mechanism	36.3	45.8	51.5	48.2

Example 3 and 4 demonstrate the results of the two accumulation mechanisms when identical impacts, in terms of their centroid values, are received, but the impacts have indices of fuzziness. In Example 3, fuzzy sets A and B have centroids $xc_A = 16.5$ and $xc_B = 37.5$ and B is a crisp set with fuzziness 0 (Figure 15 (a)). In Example 4, fuzzy set B changes to a triangular fuzzy set with the maximum fuzziness 0.5 (Figure 15 (b)). In both examples, fuzziness of fuzzy set A remains the same. Both mechanisms accumulate different impact when the degree of fuzziness of fuzzy set B is increased (Table II); 48.7 and 49.1 using the standard mechanism and 51.5 and 48.2 using the proposed mechanism when $s=4$, in Example 3 and 4, respectively. However, the change in the accumulated value obtained by the standard mechanism is negligible.



(a) Definitions of fuzzy sets A ($\hat{f}(A)=0.286$) and B ($\hat{f}(B)=0$)

(b) Definitions of fuzzy sets A ($\hat{f}(A)=0.286$) and B ($\hat{f}(B)=0.5$)

Fig. 15 Definitions of fuzzy sets A and B

The new mechanism is much more sensitive to changes in fuzziness of fuzzy set B. Figure 16 presents the accumulation of impacts when fuzziness of fuzzy sets B changes from 0 to

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

0.5. It can be observed that when the standard mechanism is used, the accumulated value abruptly increases when the degree of fuzziness of fuzzy set B reaches the value around 0.45. It is because the size of the area of the fuzzy set representing the greater variation, fuzzy set B, is smaller than the size of the area of fuzzy set A representing the smaller variation, when the fuzziness of B is below 0.45; therefore, the standard mechanism does not accumulate impacts correctly.

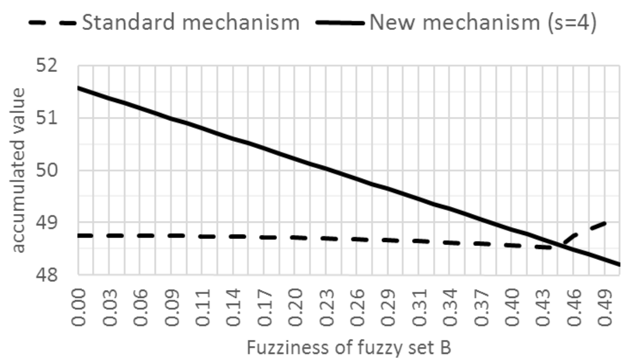


Fig. 16 Comparison of results of the two mechanisms for accumulation of impacts in Example 3

(3) Computational complexity

The new accumulation of impacts mechanism is considerably less complex than the standard reasoning mechanism. It requires 6 simple arithmetic operations to be performed (equations (21) to (25)). On the other hand, the standard mechanism requires performing at least 20 operations for every fuzzy set which is accumulated, resulting in a high number of calculations required (equations (18) to (20)). This is due to the recursive nature of the standard mechanism that need to be performed for all the points on the universe of discourse, where two accumulated fuzzy sets overlap. The differences between both mechanisms are summarized in the Table III. For example, the RBFCM developed for the cyber defence case study, consists of 31 nodes and 44 relationships between them. Running the model for 20 iterations took 1.2 seconds on the core i5 powered laptop.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

TABLE III

COMPARISON OF NUMBER OF OPERATIONS REQUIRED TO CALCULATE THE ACCUMULATED IMPACT

	Standard	New
Calculation of the shape of VOS	Minimum of 20 operations per accumulated fuzzy set	3
Calculation of the centroid	2 (centroid of VOS_+ and VOS_-)	2
Defuzzification	1	1

VI. COMPLEX RELATIONSHIPS

In some cases, to determine an impact on the effect node, information about the state of more than one causal node should be considered simultaneously, rather than separately.

Let us consider a complex relationship cFCR presented in Figure 1, where Ability to detect compromises to CIS depends on the Understanding of Cyber tools and Cyber tradecraft. While the relationship between these three nodes could be modelled using two FCR relationships, it would not capture the nuanced character of this relationship. An increase in understanding of Cyber tools needs to be followed by an increase in understanding of Cyber tradecraft to have a positive impact on the Ability to detect compromises to CIS. If two separate FCR relationships are used, then any positive change in one of the nodes Understanding of Cyber tools or Understanding of Cyber tradecraft would have a positive impact on the Ability to detect compromises to CIS. However, according to cyber defence experts, it would not represent the reality where knowledge about the states of the two nodes simultaneously is required to define the impact.

Generally, a cFCR consists of fuzzy IF-THEN rules whose antecedent part includes all the causal nodes involved in the relationship, joined with AND operator. Rules are defined as follows:

R_i: IF X is A_j AND Y is B_j THEN Z is C_j

where R_i is the i^{th} rule, i=1,…,$N \times N$, X, Y, Z are nodes involved in the relationship and A_j, B_j, C_j, j=1…N, are fuzzy sets defining these nodes.

While in the case of FCR, up to two rules can be fired, in the case of cFCR up to four rules can be fired. To calculate COS_U, when several, complex rules are fired it is necessary to calculate the union U of the "cut" consequent sets of the effect node:

$U(z) = \max[\mathrm{C'}_1(z),...,\mathrm{C'}_i(z),...,\mathrm{C'}_{N\times N}(z)]$

where, $\mathrm{C'}_i(z) = m_i \cdot \mathrm{C}_i(z)$, i=1,...,$N \times N$ is the "cut" consequent fuzzy set and

$m_i = m_{A_j} \cdot m_{B_j}$, i=1,...,$N \times N$ is the i^{th} rule firing level.

Using an extension of the reasoning mechanism proposed in Section IV.B, COS_U is calculated in such a way that its core, inner and outer bases depend on the shape of all fuzzy sets C_i, i=1,…,$N \times N$:

$$core_{COS_U} = c\sum_{i=1}^{N\times N}\frac{core_{C_i}}{dist_{C_i}} \tag{28}$$

$$bi_{COS_U} = c\sum_{i=1}^{N\times N}\frac{bi_{C_i}}{dist_{C_i}} \tag{29}$$

$$bo_{COS_U} = c\sum_{i=1}^{N\times N}\frac{bo_{C_i}}{dist_{C_i}} \tag{30}$$

$$support_{COS_U} = bi_{COS_U} + core_{COS_U} + bo_{COS_U} \tag{31}$$

where $c = \dfrac{1}{\sum_{i=1}^{N\times N}\dfrac{1}{dist_{C_i}}}$

$dist_{C_i} = | xc_U - xc_{C_i} |$, is the distance between defuzzified union of "cut" fuzzy sets U and fuzzy set C_i, where only those fuzzy sets whose firing level is greater than 0 are considered, $m_i > 0,\ i = 1,..., N \times N$.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

Once the characteristics of COS_U are obtained, it is determined in such a way that $xc_{COS_U} = xc_U$.

An example of determining union U resulting from firing four rules $\mathrm{R_i, R_j, R_k}$ and $\mathrm{R_l}$ is presented in Figure 17 where:

R_i: IF X is $\mathrm{A}_i(m_{A_i})$ AND Y is $\mathrm{B}_i(m_{B_i})$ THEN Z is $\mathrm{C}_i(m_{C_i})$
R_j: IF X is $\mathrm{A}_j(m_{A_j})$ AND Y is $\mathrm{B}_j(m_{B_j})$ THEN Z is $\mathrm{C}_j(m_{C_j})$
R_k: IF X is $\mathrm{A}_k(m_{A_k})$ AND Y is $\mathrm{B}_k(m_{B_k})$ THEN Z is $\mathrm{C}_k(m_{C_k})$
R_l: IF X is $\mathrm{A}_l(m_{A_l})$ AND Y is $\mathrm{B}_l(m_{B_l})$ THEN Z is $\mathrm{C}_l(m_{C_l})$

$A_i = A_j, A_k = A_l, B_i = B_k, B_j = B_l$ and C_i, C_j, C_k and C_l are different fuzzy sets.

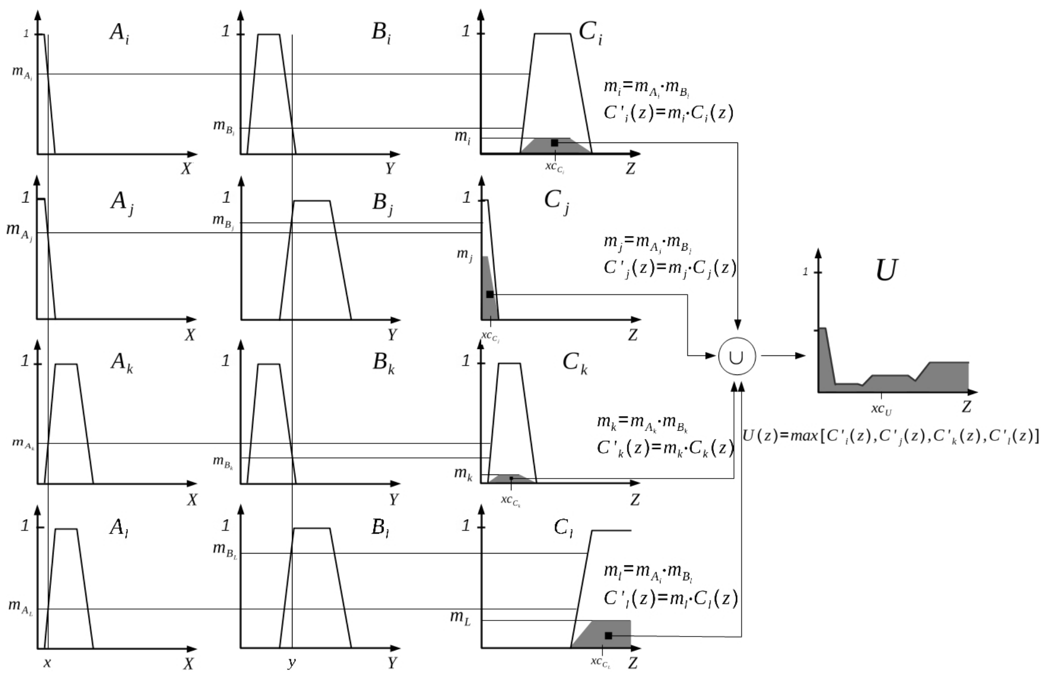


Fig. 17 Fuzzy reasoning for a complex FCR

As it can be seen, because of inputs x and y being received by Nodes X and Y, rules defining the complex relationship between nodes X, Y and Z are fired with the following firing levels:

$$m_{A_j} = m_{A_i}$$
$$m_{A_k} = m_{A_l} = 1 - m_{A_i}$$
$$m_{B_k} = m_{B_i}$$
$$m_{B_l} = m_{B_j} = 1 - m_{B_i}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2017.2686363, IEEE Transactions on Fuzzy Systems

Firing levels of the consequent fuzzy sets, defining Node Z, are resulting from calculating the product of firing levels of fuzzy sets defining nodes X and Y:

$$m_i = m_{A_i} \cdot m_{B_i}$$
$$m_j = m_{A_j} \cdot (1 - m_{B_i})$$
$$m_k = (1 - m_{A_i}) \cdot m_{B_i}$$
$$m_l = (1 - m_{A_i}) \cdot (1 - m_{B_i})$$

The sum of firing levels of consequents of Node Z, $m_i + m_j + m_k + m_l$ is equal to 1.

CONCLUSIONS

FCMs have proven its usefulness in analysis of systems which are based on qualitative knowledge. A new prospect has been opened with introduction of RBFCMs, with a novel reasoning and accumulation of impacts mechanisms. In this paper, we proposed to eliminate a constraint on how membership functions of node variations in RBFCMs have to be defined; the higher the variation represented by the fuzzy set, the bigger the area of the corresponding membership function. The new reasoning and accumulation mechanisms, we propose, take into consideration the standard semantics of fuzzy sets, where associated uncertainty is measured by fuzziness. Such an improvement provides more flexibility and use of intuition when modelling using RBFCMs. Experts are no longer bound to using the predefined membership functions that satisfy requirements of the standard RBFCM’s mechanisms. The new accumulation of impacts mechanism is commutative and associative and significantly simpler when compared to recursive algorithm used in the standard accumulation mechanism. In addition, new complex FCR are proposed, which can help in capturing complexity of real world systems. The RBFCM proposed is more complex than FCMs in terms of computational power that is needed to run simulation. However, this demand is small considering modelling capabilities it offers compared to FCM.

Further developments of RBFCM are focused on several directions. The mechanism for accumulation of impacts can be modified in such a way as to generate one fuzzy set which represents accumulation, rather than two positive and negative ones. This would enable easier interpretation of the impact accumulation. The time component embedded in FCR is being further developed to improve the representation of dynamics in RBFCMs. The reasoning mechanism has been further modified to operate with loops in RBFCMs. A very important element of RBFCMs, that has not received enough attention so far, is learning of membership functions and/or rules between concepts. FCS learning methods will be analysed to be adapted to incorporate into the accumulation mechanism proposed.

References

[1] B. Kosko, “Fuzzy Cognitive Maps”, *International Journal of Man-Machine Studies*, vol. 24, pp. 65-75, 1986.

[2] E. H. Mamdani, “Application of fuzzy logic to approximate reasoning using linguistic synthesis”, *IEEE Transactions on Computers*, vol. 26, pp. 1182–1191, 1977.

[3] J. Tome, J. P. Carvalho, “Rule Based Fuzzy Cognitive Maps: Fuzzy Causal Relations,” Computational Intelligence for Modelling, Control and Automation, 1999.

[4] J. Tome, M. Carola, J. P. Carvalho, “Forest Fire Modelling using Rule-Based Fuzzy Cognitive Maps and Voronoi Based Cellular Automata,” in Proc. of the Annual Conference of the North American Fuzzy Information Processing Society, 2006.

[5] J. Tome, J. P. Carvalho, “Rule Based Fuzzy Cognitive Maps in Socio-Economic Systems,” in Proc. of IFSA-EUSFLAT, 2009.

[6] L. Wise, A. G. Murtra, J. P. Carvalho, M. Mesquita, “Qualitative modelling of fishermen’s behavior in a pelagic fishery,” in Ecological Modelling, vol. 228, pp. 112-122, 2012.

[7] L. Wise, P. Fonseca, A. G. Murta, C. Silva, H. Mendes, J.P. Carvalho, M. D. F. Borges, A. Campos, "A knowledge-based model for evaluating the impact of gear-based management measures under Europe's new Common Fisheries Policy," in Proc. of the ICES Journal of Marine Science, 2015.

[8] P. Zdanowicz, D. Petrovic, C. Irwin, "Modelling and Analysis of Defense Lines of Developments Using Fuzzy Causal Maps with a Practical Example," in Proc. of the 30th International Symposium on Military OR, 2013.

[9] A. Peña-Ayala, J. H. Sossa-Azuela, "Decision making by rule-based fuzzy cognitive maps: An approach to implement student-centered education," Intelligent Systems Reference Library, vol. 54, pp. 107-120, 2014.

[10] R. E. Bellman, L. A. Zadeh, "Decision-making in a fuzzy environment," Management Science, vol. 17, pp. 141-164, 1970.

[11] J. P. Carvalho, "Interpolated Linguistic Terms," Proceedings of the Fuzzy Information, Processing NAFIPS '04. IEEE Annual Meeting, vol. 1, pp. 151-156, 2004.

[12] G. J. Klir, T. A. Folger, Fuzzy sets, uncertainty and information. Upper Saddle River, NJ: Prentince-Hall International, Inc., 1988.

[13] A.C. Thompson, Minkowski Geometry, Encyclopedia of Mathematics and its Applications. Cambridge University Press, Cambridge, 1996.

[14] J.P. Carvalho, J. Tome, "Fuzzy mechanisms for qualitative causal relations," Studies in Fuzziness and Soft Computing, vol. 243, pp. 393-415, 2009.

[15] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," IEEE Transactions on Systems, Man and Cybernetics , vol. 15, no. 1, pp. 116–132, 1985.

[16] C.C. Lee, "Fuzzy logic in control systems: fuzzy logic controller. II," IEEE Transactions on Systems, Man and Cybernetics, vol.20, pp. 419-435, 1990.

[17] E. I. Papageorgiou, "Learning Algorithms for Fuzzy Cognitive Maps—A Review Study," IEEE Transactions on Systems, Man, and Cybernetics, vol. 42, no. 2, pp. 150-163, 2012.

[18] E. Papageorgiou, C. D. Stylios, P. P. Groumpos, “Active Hebbian learning algorithm to train fuzzy cognitive maps,” Int. J. App. Reasoning, vol. 37, no. 3, pp. 219–249, 2004.

[19] S. Pal, A. Konar, “Cognitive Reasoning using Fuzzy Neural Nets,” IEEE Trans. on Systems, Man and Cybernetics, vol. 26 no. 4, pp. 616-619, 1996.

[20] G. Acampora, W. Pedrycz, and A. Vitiello, “A Competent Memetic Algorithm for Learning Fuzzy Cognitive Maps,” IEEE Trans. on Fuzzy Systems, 23(6), pp. 2397-2411, 2015.

[21] J. Liu, Y. Chi, and C. Zhu, “A dynamic multi-agent genetic algorithm for gene regulatory network reconstruction based on fuzzy cognitive maps”, IEEE Trans. on Fuzzy Systems, 24(2), pp. 419-431, 2016.

[22] C. F. Juang, C. W. Hung and C. H. Hsu, "Rule-Based Cooperative Continuous Ant Colony Optimization to Improve the Accuracy of Fuzzy System Design," in IEEE Transactions on Fuzzy Systems, vol. 22, no. 4, pp. 723-735, Aug. 2014.

[23] M. Antonelli, P. Ducange and F. Marcelloni, "Genetic Training Instance Selection in Multiobjective Evolutionary Fuzzy Systems: A Coevolutionary Approach," in IEEE Transactions on Fuzzy Systems, vol. 20, no. 2, pp. 276-290, April 2012.